Electric and Magnetic Coupling through Small Apertures in Shield Walls of Any Thickness

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Abstract—A method is presented for evaluating the coupling between two identical resonant cavities coupled by a small aperture in a plane common wall of arbitrary thickness. The coupling is related to the frequencies of the symmetric and asymmetric modes of oscillation of the coupled cavity structure, and a variational technique is used to determine those frequencies.

The method is applied to circular and rectangular apertures, and it is shown that the coupling is separable into electric and magnetic terms. The results enable theoretical solutions to be obtained for the electric and magnetic polarizabilities of circular and rectangular apertures in walls of zero thickness, and equivalent polarizabilities to be obtained when the wall thickness is nonzero. Curves of numerical values are given for circular and rectangular apertures. With zero wall thickness, the results obtained are the same as those of Bethe for a circular aperture and give good agreement with Cohn's experimental results for rectangular apertures.

I. Introduction

THE determination of the field coupled through a small aperture in a common wall between two regions is important in the design of such items as waveguide directional couplers and coupled resonator filters. Bethe [1] investigated the coupling through a small circular aperture in a conducting plane wall of zero thickness, and his solution utilizing equivalent electric and magnetic dipole moments has been extensively used in the design of coupled cavity and waveguide systems. Bethe's method of solution is applicable to small elliptical apertures as well as to circular apertures, but not to rectangles or more complicated geometrical figures. Cohn [2], [3] developed an electrolytic tank method for measuring the electric and magnetic polarizabilities of small apertures of arbitrary shape, and presented data for a selection of apertures.

The work of Bethe and the experimental results of Cohn apply to small apertures in walls of zero thickness. The effect of a finite wall thickness is often approximated by including the attenuation of an evanescent waveguide mode traversing the wall thickness [4]–[6].

The evaluation of the coupling through apertures in plane walls of arbitrary thickness, for apertures of transverse dimensions (i.e., at right angles to the wall thickness) small in wavelengths, is considered here. The aperture is not close to a corner or other discontinuity, and the solutions are obtained by making a number of approximations that are exact in the limit as the aperture transverse dimensions go to zero. For an aperture in a wall of finite thickness, the method allows the aperture to be filled with an arbitrary lossless isotropic material.

Only the case of coupling between two lossless, symmetrically oriented, identical resonant cavities is considered. For other applications the aperture polarizabilities may be used.

A time dependence of the form $e^{j\omega t}$ is implied throughout. The method is presented in outline only. The details are given in [7].

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II. Relationship between Aperture Polarizabilities and Coefficient of Coupling

The theory developed originally by Bethe [1] for circular apertures, and later generalized by Collin [8], shows that if a field exists on one side of a conducting plane wall of zero thickness, and a small aperture (aperture dimensions $\ll\lambda$) is then cut in the wall, the field in the second region is the same as that from an electric dipole $\boldsymbol{P}_0$ normal to the wall and a magnetic dipole $\boldsymbol{M}_0$ tangential to the wall, both at the center of the aperture with the aperture closed.

Quantitatively, the electric and magnetic dipole moments are given by

$$\begin{aligned} \boldsymbol{P}_0 &= -\epsilon_0 p_e \boldsymbol{E}_n \\ \boldsymbol{M}_0 &= -p_m \boldsymbol{H}_t \end{aligned} \tag{1}$$

where p_e and p_m are the electric and magnetic polarizabilities of the small aperture and $\boldsymbol{E}_n$ and $\boldsymbol{H}_t$ are the vector normal electric field and the tangential magnetic field, respectively, at the aperture location prior to opening the aperture. The vector normal electric field $\boldsymbol{E}_n$ may be defined as equal to $\hat{n}(\hat{n}\cdot\boldsymbol{E})$ where $\hat{n}$ is a unit normal vector. It is implicit in (1) that $\boldsymbol{H}_t$ is parallel to a principal axis of the aperture. If this is not the case, then $\boldsymbol{H}_t$ has to be separated into two components each parallel to a principal axis, and the resultant magnetic dipole moment obtained from two components.

If the aperture polarizability concept is applied to the coupling between two lossless, symmetrically oriented, identical resonant cavities having a common plane wall of zero thickness and containing a small aperture, then the coefficient of coupling K between the two cavities is related to the electric and magnetic polarizabilities of the coupling aperture by

$$K = p_m \frac{\boldsymbol{H}_{pt} \cdot \boldsymbol{H}_{pt}}{\iiint_v \boldsymbol{H}_p \cdot \boldsymbol{H}_p \, dv} + p_e \frac{\boldsymbol{E}_{pn} \cdot \boldsymbol{E}_{pn}}{\iiint_v \boldsymbol{E}_p \cdot \boldsymbol{E}_p \, dv} \qquad (2)$$

where $\boldsymbol{H}_p$ is the predominant magnetic field eigenfunction in the cavity, $\boldsymbol{E}_p$ is the electric field corresponding to $\boldsymbol{H}_p$, and $\boldsymbol{H}_{pt}$ and $\boldsymbol{E}_{pn}$ are the tangential magnetic field and the vector normal electric field at the aperture location corresponding to $\boldsymbol{H}_p$ and $\boldsymbol{E}_p$, respectively. The normalizing integrations in the denominators of (2) are over the volume v of one cavity. Equation (2) is a generalization of the expression given by Matthaei *et al.* [9] for the case of magnetic coupling only.

For a circular aperture of radius R in an infinitely thin wall, Bethe's results for p_e and p_m are

$$p_e = -\frac{2}{3} R^3$$

and

$$p_m = \frac{4}{3} R^3.$$

III. Aperture in a Wall of Arbitrary Thickness

A. The Equivalent Network and its Properties

Two identical resonant cavities coupled by a small aperture in a plane common wall are shown in Fig. 1. The interior of each half of the structure is divided into two regions, a "cavity region" and an "aperture region," by a plane surface s.

The aperture region is uniformly filled with a lossless material with electrical characteristics $\mu_0\mu_r$ and $\epsilon_0\epsilon_r$, and the cavity regions are assumed air filled (i.e., μ_0, ϵ_0).

The two cavities coupled by the small aperture will exhibit the behavior of two lightly coupled identical resonant circuits, for which a general representation is given in Fig. 2(a). In Fig. 2(b), the coupling reactance X_c has been separated into two parts.

A coupled cavity structure such as that in Fig. 1 has two oscillation states for each cavity resonance. One oscillation state corresponds to an electric wall boundary condition on the symmetry plane in Fig. 1 or a short circuit on the symmetry line y–y' of Fig. 2(b). The other oscillation state corresponds to a magnetic wall boundary condition on the symmetry plane in Fig. 1 or an open circuit on the symmetry line y–y' in Fig. 2(b). The oscillation frequencies of the real structure and the equivalent circuit must be the same under the corresponding boundary conditions.

It may be shown that the coefficient of coupling K of the equivalent circuit, and also by analogy of the coupled cavity resonators, is expressible in the form

$$K = \frac{\omega_s^2 - \omega_o^2}{\omega_s^2 + \omega_o^2} \qquad (3)$$

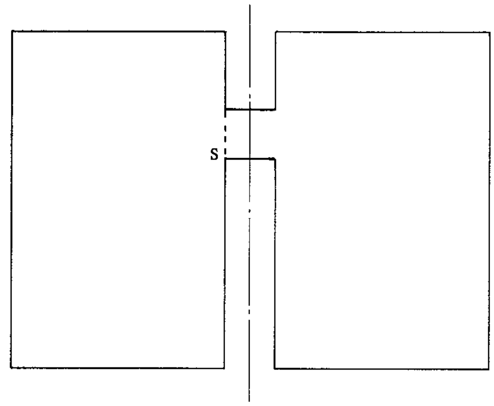

Fig. 1. Two identical cavities coupled by an aperture in a plane common wall.

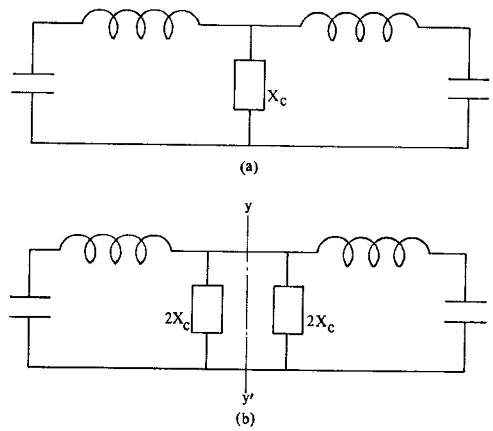

Fig. 2. Network representation of coupled cavities. (a) Single coupling reactance. (b) Coupling reactance separated into two parts.

where ω_s is the angular oscillation frequency in the short-circuit or electric wall case and ω_o is the angular oscillation frequency in the open-circuit or magnetic wall case. In the determination of the resonant frequencies of the aperture coupled cavities, only one-half of the structure need be considered.

B. Variational Determination of the Resonant Frequencies

One of several alternative viewpoints for obtaining a variational solution for the resonant frequencies is Rumsey's "reaction concept" [10]. The method used is the same in principle for both symmetry plane boundary conditions.

If the tangential $\boldsymbol{E}$ field on the surface s is postulated and for both the aperture region and the cavity region the consequent tangential magnetic fields are determined as functions of ω, then the equation

$$\iint_s \boldsymbol{E} \times \boldsymbol{H}_a \cdot d\boldsymbol{S} = \iint_s \boldsymbol{E} \times \boldsymbol{H}_c \cdot d\boldsymbol{S} \qquad (4)$$

where the subscripts a and c denote the aperture and cavity regions, respectively, is an implicit variational equation for the resonant frequency. This is because the only source necessary to make a self-consistent solution of Maxwell's equations is an electric-current sheet on s

given by $J_s = \hat{n} \times (H_c - H_a)$, and thus (4) is obtained by equating the self-reaction $\iiint E \cdot J\, dv$ to zero [11], [12].

In the application of this result, the postulated tangential E field on s contains a number of adjustable amplitude coefficients, for which the optimum values are obtained by the Ritz procedure. Because the E field of a single mode in a lossless resonant region has everywhere the same phase, the postulated tangential E on s can be restricted to real functions. H_a and H_c will as a consequence be imaginary.

C. Field in the Cavity Region

The magnetic field H_c in the cavity region can be expanded in a complete set of orthogonal eigenfunctions of two types, i.e.,

$$H_c = \sum_i h_i H_i + \sum_i g_i G_i$$

where h_i and g_i are the amplitude coefficients of the H_i and G_i eigenfunctions and the infinite number of H_i functions correspond to the magnetic fields of the natural modes of oscillation of the region [13], [14].

For H_c imaginary, the H_i may be taken as imaginary and the h_i as real. If the H_i are normalized according to

$$\mu_0 \iiint_v H_i \cdot H_i \, dv = -1 \qquad (5)$$

then with a tangential E specified on the surface s and zero tangential E on the rest of the boundary of the cavity region, the amplitude coefficient h_i of an arbitrary H_i eigenfunction can be determined as

$$h_i = -\frac{j\omega}{\omega_i^2 - \omega^2} \iint_s E \times H_i \cdot dS \qquad (6)$$

where ω_i is the resonant frequency corresponding to the H_i eigenfunction.

When a small aperture is cut in the wall between the two cavities and an electric or magnetic wall is placed on the symmetry plane, the resonant frequency is moved very little from the oscillation frequency of the cavity. Therefore, one of the $h_i H_i$ terms in the expansion of H_c is strongly dependent on ω and is much greater than any of the other terms. Denoting this predominant eigenfunction by H_p with corresponding h_p and ω_p, let

$$H_c = h_p H_p + H_f \qquad (7)$$

where the magnetic field H_f is the sum of the $h_i H_i$ and $g_i G_i$ terms other than $h_p H_p$. This "fringing" field H_f exists primarily in the vicinity of the aperture. Then (6) and (7) can be used to obtain

$$\iint_s E \times H_c \cdot dS = -\frac{j\omega}{\omega_p^2 - \omega^2} \left\{ \iint_s E \times H_p \cdot dS \right\}^2 + \iint_s E \times H_f \cdot dS. \qquad (8)$$

An analog of the right side of (8) is the network in Fig. 3, with the resonant arm corresponding to the predominant

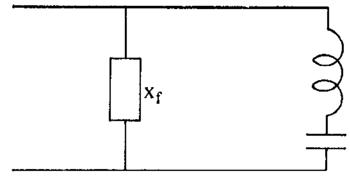

Fig. 3 Network with resonant arm corresponding to predominant eigenfunction, and reactance X_f corresponding to all other eigenfunctions.

H_p eigenfunction and the reactance X_f being associated with H_f.

The fringing magnetic field H_f can in principle be determined from the eigenfunction expansion as an infinite series. However, an alternative and simpler approach is to make a reasonable approximation to the $\iint_s E \times H_f \cdot dS$ term. In Fig. 3 the reactance X_f is a slowly varying function of frequency, and it would be the off-resonance reactance if the contribution of the particular H_p eigenfunction to the off-resonance field was negligible.

The off-resonance field is a quasi-static field localized about the small aperture. If it is expanded in a series of eigenfunctions orthogonal over the cavity volume, then as the aperture size decreases the field becomes more localized, more eigenfunctions are required for an adequate representation, and the proportionate contribution of each particular eigenfunction, such as H_p, becomes smaller. This suggests that the reactance X_f is indeed the off-resonance reactance. However, the off-resonance reactance of a small aperture in a plane cavity wall not close to other boundary surfaces is essentially the same as if the wall was infinite in extent.

A suitable approximation for $\iint_s E \times H_f \cdot dS$ therefore is the corresponding quantity for an aperture in an infinite plane conducting wall. Then $\iint_s E \times H_c \cdot dS$ is given by (8) with this approximation used to obtain $\iint_s E \times H_f \cdot dS$.

D. Solution for the Coefficient of Coupling K

Equation (8) may be combined with (4) to give

$$\frac{\omega_p^2 - \omega^2}{2\omega^2} = -\frac{j}{2\omega} \frac{\left\{ \iint_s E \times H_p \cdot dS \right\}^2}{\iint_s E \times (H_a - H_f) \cdot dS}. \qquad (9)$$

Two frequency shift parameters S_0 and S_s are now defined as

$$S_0 = \frac{\omega_p^2 - \omega_0^2}{2\omega_0^2} \qquad (10)$$

$$S_s = \frac{\omega_p^2 - \omega_s^2}{2\omega_s^2}. \qquad (11)$$

Then, from (3), (10), and (11),

$$K = \frac{S_0 - S_s}{1 + S_0 + S_s}.$$

As the aperture transverse dimensions go to zero, S_0 and

S_s go to zero. Thus

$$K \to S_0 - S_s.$$

Therefore, if the right side of (9) is evaluated by using the best possible approximation to the true tangential $\boldsymbol{E}$ field on the surface s, obtained by using a trial field with a large number of adjustable parameters, then the coefficient of coupling K is given by the difference between the values obtained with the magnetic and electric wall boundary conditions.

If the aperture cross section is of simple shape, such as circular or rectangular, a suitable and complete set of trial functions for the tangential $\boldsymbol{E}$ field is a sum of all of the $\boldsymbol{E}$ fields of the below-cutoff waveguide modes in the aperture region.

E. Separation of S_0 and S_s into Independent Parts

Consider the postulated tangential $\boldsymbol{E}$ field on the surface s to be the sum of two terms, e.g., $\boldsymbol{E}=\boldsymbol{E}^{\mathrm{I}}+q\boldsymbol{E}^{\mathrm{II}}$ where q is a proportionality coefficient. Then corresponding to $\boldsymbol{E}^{\mathrm{I}}$ there will be $\boldsymbol{H}_a^{\mathrm{I}}$ and $\boldsymbol{H}_f^{\mathrm{I}}$, and corresponding to $q\boldsymbol{E}^{\mathrm{II}}$ there will be $q\boldsymbol{H}_a^{\mathrm{II}}$ and $q\boldsymbol{H}_f^{\mathrm{II}}$. If such an $\boldsymbol{E}$ field is used to determine either of the frequency shift parameters S_0 or S_s from (9), then in accordance with the variational nature of the solution, q is to be adjusted so that the resonant frequency is stationary with respect to q. For arbitrary $\boldsymbol{E}^{\mathrm{I}}$ and $\boldsymbol{E}^{\mathrm{II}}$ field functions, the resultant expression for S_0 or S_s is dependent on the forms of both $\boldsymbol{E}^{\mathrm{I}}$ and $\boldsymbol{E}^{\mathrm{II}}$. If, however, $\boldsymbol{E}^{\mathrm{I}}$ and $\boldsymbol{E}^{\mathrm{II}}$ on s together with their associated magnetic fields satisfy the particular "independence condition"

$$\iint_s \boldsymbol{E}^{\mathrm{II}} \times (\boldsymbol{H}_a^{\mathrm{I}} - \boldsymbol{H}_f^{\mathrm{I}})\cdot d\boldsymbol{S} + \iint_s \boldsymbol{E}^{\mathrm{I}} \times (\boldsymbol{H}_a^{\mathrm{II}} - \boldsymbol{H}_f^{\mathrm{II}})\cdot d\boldsymbol{S} = 0$$

and the optimum value of q is determined, it is found that the result for S_0 or S_s is simply the sum of the two results that would be obtained by using $\boldsymbol{E}^{\mathrm{I}}$ and $\boldsymbol{E}^{\mathrm{II}}$ as separate trial fields. If both S_0 and S_s are separable in this manner, then the coefficient of coupling is also separable into two independent terms. If either of the two fields can be further subdivided in a manner that satisfies the independence condition, then the number of separate contributions is not limited to two.

IV. A Small Circular Aperture

A. General Aspects

A combined Cartesian and cylindrical coordinate system x, y, z, r, θ is established as shown in Fig. 4, with the z axis lying on the axis of the aperture region and the $z=0$ plane at the surface s separating the aperture region from the cavity region.

The aperture radius is R and the ratio of (wall thickness)/(aperture radius) is designated T_R. The aperture region contains a lossless material of relative permeability μ_r and relative permittivity ϵ_r.

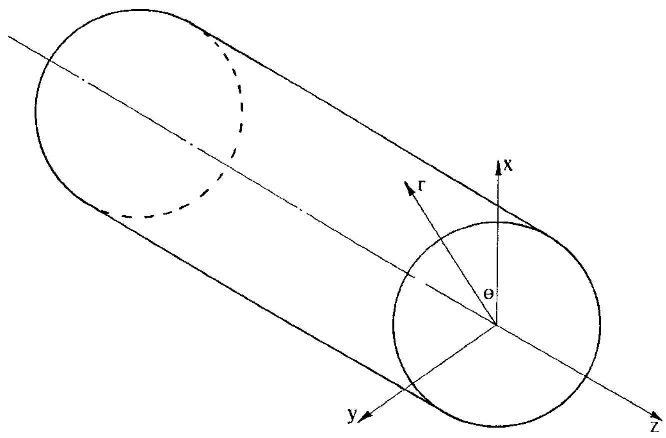

Fig. 4. Coordinate system for circular aperture. The z axis is along the aperture axis and the $z=0$ plane is at the wall surface.

B. Mode Fields Contributing to S_0 and S_s

Of all the possible below-cutoff circular waveguide modes in the aperture region, it is necessary to determine which ones are significant in the evaluation of S_0 and S_s, and thereby contribute to K. From (9), mode fields will be significant only if the corresponding $\iint_s \boldsymbol{E}\times\boldsymbol{H}_p\cdot d\boldsymbol{S}$ is nonzero, i.e., if

$$\iint_s (E_r H_{p\theta} - E_\theta H_{pr})\, r dr d\theta \neq 0. \tag{12}$$

The tangential $\boldsymbol{H}_p$ field on s is separated into its x and y components, and each is expanded in a Taylor expansion about the center of s. Only terms up to and including first derivatives are retained. A conversion to polar coordinates gives H_{pr} and $H_{p\theta}$. For TM_{lm} mode $\boldsymbol{E}$ fields, (12) is found to be satisfied only if $l=0$.

For TE_{lm} mode $\boldsymbol{E}$ fields, (12) is satisfied for $l=0$, 1, and 2. However, the contributions of the TE_{0m} and TE_{2m} mode fields to S_0 and S_s contain the factor R two powers higher than do the contributions of the TE_{1m} mode field. As $R\to 0$, therefore, the TE modes of significance are the TE_{1m} modes for all m.

Because of trigonometric orthogonality, the TM_{0m} modes and the TE_{1m} modes form separable sets in accordance with the independence condition of Section III-E, and the coefficient of coupling K is the sum of the results obtained by considering each set separately.

C. TM_{0m} Modes in Aperture Region

With a sum of TM_{0m} mode fields, i.e., TM_{01}, TM_{02}, TM_{03}, $\cdots$, with arbitrary amplitude coefficients, first is determined the fringing field for the aperture in an infinite plane conducting wall. The procedure is the same in principle as that given by Harrington for two-dimensional situations [12, pp. 180–186]. The scalar potential is expressed as an integral over all possible solutions of the scalar Helmholtz equation, and this integral is then interpreted as a Hankel transform. The tangential $\boldsymbol{E}$ field components at the aperture are known, and the scalar potential is thereby defined.

The evaluation of $\iint_s \boldsymbol{E}\times\boldsymbol{H}_a\cdot d\boldsymbol{S}$, i.e., on the aperture side of s, is carried out using straightforward procedures for circular waveguide modes. This term is dependent on the symmetry-plane boundary condition. Thus S_0 and S_s can be obtained from (9) in terms of $\boldsymbol{H}_p$ or its spatial derivatives. The variational nature of the solution for the corresponding resonant frequency (ω_0 or ω_s) gives a basis for optimizing the mode amplitude coefficients in each case.

Then the coefficient of coupling K^{TM} due to the TM_{0m} modes is found to be of the form

$$K^{TM} = -C_E R^3 \epsilon_0 [E_{pz}]_c^2$$

where $[\,]_c$ denotes evaluation at the center of s. The dimensionless coefficient C_E is a function of T_R and ϵ_r, and $\boldsymbol{E}_p$ is the electric field corresponding to $\boldsymbol{H}_p$. $\boldsymbol{E}_p$ arises from terms $((\partial H_{py}/\partial x)-(\partial H_{px}/\partial y))$ in the Taylor expansion of $\boldsymbol{H}_p$, and its normalizing to be consistent with (5) is

$$\epsilon_0 \iiint_v \boldsymbol{E}_p\cdot\boldsymbol{E}_p\, dv = 1. \tag{13}$$

D. TE_{1m} Modes in Aperture Region

The procedure with the TE_{1m} modes is similar to that for the TM_{0m} modes. It is convenient to choose the coordinate system orientation such that $[H_{px}]_c$ is zero, and the result obtained is of the form

$$K^{TE} = -C_H R^3 \mu_0 [H_{py}]_c^2$$

in which the dimensionless coefficient C_H is a function of T_R and μ_r.

E. Total Coupling

The total coefficient of coupling K is the sum of K^{TM} and K^{TE}. If the squares of the normal $\boldsymbol{E}$ field and the tangential $\boldsymbol{H}$ field at the center of the surface s, i.e., $[E_{pz}]_c^2$ and $[H_{py}]_c^2$, are denoted by $\boldsymbol{E}_{pn}\cdot\boldsymbol{E}_{pn}$ and $\boldsymbol{H}_{pt}\cdot\boldsymbol{H}_{pt}$, respectively, and if the normalizing of $\boldsymbol{E}_{pn}$ and $\boldsymbol{H}_{pt}$ from (5) and (13) is shown explicitly, then

$$K = C_H R^3 \frac{\boldsymbol{H}_{pt}\cdot\boldsymbol{H}_{pt}}{\iiint_v \boldsymbol{H}_p\cdot\boldsymbol{H}_p\, dv} - C_E R^3 \frac{\boldsymbol{E}_{pn}\cdot\boldsymbol{E}_{pn}}{\iiint_v \boldsymbol{E}_p\cdot\boldsymbol{E}_p\, dv}. \tag{14}$$

If (14) is compared with (2), derived from the aperture polarizability viewpoint, it is seen that the concept of aperture polarizability may for a circular aperture be extended to the finite wall thickness case if the equivalent polarizabilities are obtained from

$$p_m = C_H R^3$$

and

$$p_e = -C_E R^3.$$

F. Calculation of the Numerical Coefficients C_E and C_H

A maximum of 12 TM_{0m} and TE_{1m} modes were used to determine C_E and C_H, respectively. For the particular case of zero wall thickness, the results with from 5 to 12 mode fields are shown in the "Direct" columns of Table I. As the true field has an edge singularity it cannot be represented exactly by a finite number of mode fields. However, the convergence of the numerical results can be improved by a simple polynomial extrapolation technique, which gives the values in the "Extrapolated" columns in Table I. Note the excellent agreement between the values of 0.66669 and 1.3333 for C_E and C_H, respectively, with the Bethe values of 2/3 and 4/3.

Fig. 5 gives curves of C_E and C_H against the thickness

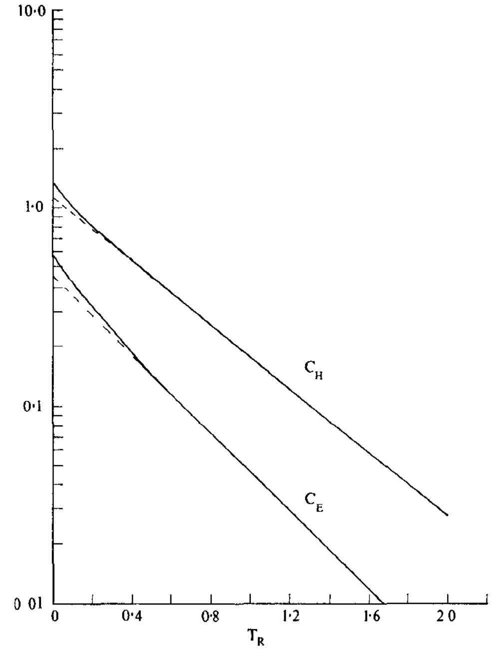

Fig. 5. C_E and C_H shown against the thickness parameter T_R for a small circular air-filled aperture ($\epsilon_r=1$, $\mu_r=1$).

TABLE I

Number of Modes	Zero Wall Thickness C_E Direct	C_E Extrapolated	C_H Direct	C_H Extrapolated
5	0.63693	0.66710	1.2781	1.3330
6	0.64195	0.66689	1.2869	1.3331
7	0.64553	0.66680	1.2933	1.3332
8	0.64820	0.66675	1.2981	1.3333
9	0.65027	0.66672	1.3019	1.3333
10	0.65193	0.66671	1.3050	1.3333
11	0.65328	0.66670	1.3075	1.3333
12	0.65441	0.66669	1.3096	1.3333

parameter T_R for a small circular air-filled aperture. These curves give the values extrapolated from the 12 mode solutions. For large T_R, C_E and C_H vary as $e^{-2.405T_R}$ and $e^{-1.841T_R}$, respectively, corresponding to the attenuations of the least attenuated associated modes, i.e., the TM_{01} and the TE_{11}. C_E and C_H approach values about 18 percent and 16 percent less than those obtained from the approximate expressions $2/3\ e^{-2.405T_R}$ and $4/3\ e^{-1.841T_R}$. Thus for thick walls ($T_R>0.5$), the equivalent electric and magnetic polarizabilities are given closely by

$$p_e \approx -0.55R^3e^{-2.405T_R}$$

$$p_m \approx 1.12R^3e^{-1.841T_R}.$$

The corresponding asymptotic straight lines are shown dashed in Fig. 5.

V. A Small Rectangular Aperture

The procedure for a small rectangular aperture in a plane wall is the same in principle as for a circular aperture. However, the rectangular-aperture case is different in that as there are two unique principal axes, there are inherent reference directions. This is of particular significance for magnetic coupling. Also, whereas a Hankel transform is used to determine the fringing field of a circular aperture, a Fourier transform is used in the rectangular-aperture case.

A rectangular coordinate system is established at the aperture as shown in Fig. 6. The $z=0$ plane is at the surface s separating the cavity region from the aperture region, and the positive z axis is in the direction into the cavity region. The transverse dimensions of the aperture are A and B, and the aspect ratio B/A is arbitrary.

The ratio of (wall thickness)/(dimension A) is designated T_A, and the aperture region is uniformly filled with a lossless material of relative permeability μ_r and relative permittivity ϵ_r.

Only the results of the analysis are given here. The coefficient of coupling K of two identical cavities coupled by a small rectangular aperture of transverse dimensions A and B in a plane common wall is

$$K = R_HA^3\frac{\boldsymbol{H}_{pt}\cdot\boldsymbol{H}_{pt}}{\iiint_v \boldsymbol{H}_p\cdot\boldsymbol{H}_p\,dv} - R_EA^3\frac{E_{pn}\cdot E_{pn}}{\iiint_v \boldsymbol{E}_p\cdot\boldsymbol{E}_p\,dv}$$

where the subscript p denotes the predominant mode field and the integrations are over the volume of one cavity.

The dimensionless coefficient R_H is associated with the TE_{mo} mode fields with m odd, and is a function of the aperture aspect ratio B/A, the wall thickness parameter T_A, and the relative permeability μ_r of the aperture region material. The dimensionless coefficient R_E is associated with the TM_{mn} mode fields with both m and n odd, and is a function of the aperture aspect ratio, the

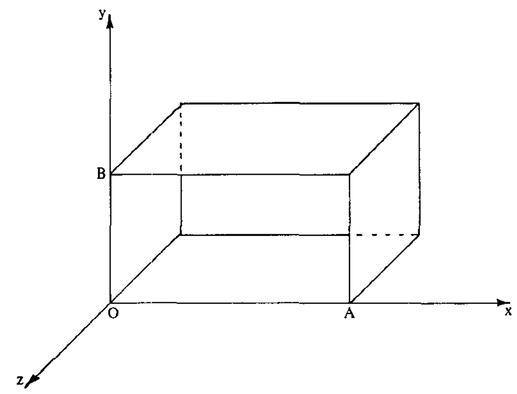

Fig. 6. Coordinate system for rectangular aperture.

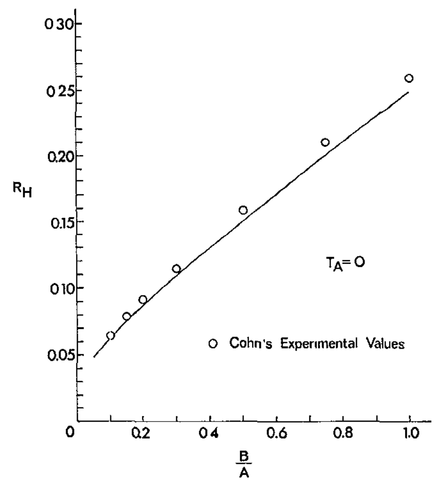

Fig. 7. R_H with zero wall thickness for aspect ratios in the range 0.05 to 1.0.

wall thickness parameter T_A, and the relative permittivity ϵ_r of the aperture region material.

The equivalent polarizabilities of a small rectangular aperture are

$$p_m = R_HA^3.$$

and

$$p_e = -R_EA^3.$$

In the determination of R_H and p_m, but not R_E and p_e, it is implied that the tangential magnetic field $\boldsymbol{H}_{pt}$ is parallel to the dimension A of the aperture.

Note that R_H and R_E relate to a *R*ectangular aperture while C_H and C_E relate to a *C*ircular aperture.

Fig. 7 gives R_H with zero wall thickness for aspect ratios B/A in the range 0.05 to 1.0. Also shown in Fig. 7 are Cohn's [2] experimental values obtained with an electrolytic tank. Fig. 8 gives R_H with zero wall thickness for aspect ratios from 1.0 to 10.0.

Fig. 9 gives R_E with zero wall thickness for aspect ratios B/A up to 1.0. For electric coupling there is freedom to choose which side of the rectangle is dimension A, and therefore Fig. 9 can be used for all aspect

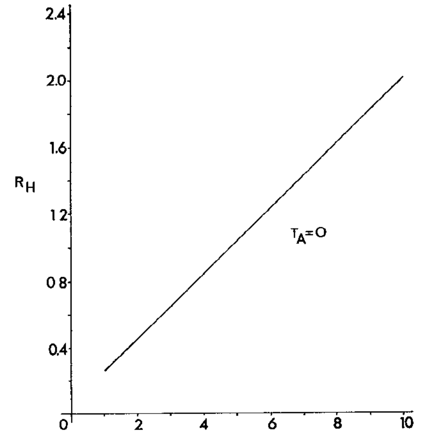

Fig. 8. R_H with zero wall thickness for aspect ratios in the range 1.0 to 10.0.

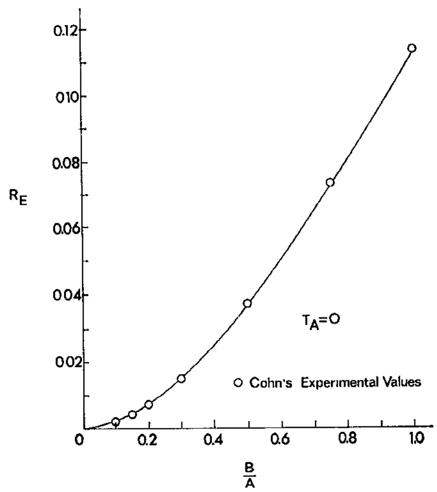

Fig. 9. R_E with zero wall thickness for aspect ratios up to 1.0.

ratios. Cohn's [3] electrolytic tank experimental values for R_E are also shown in Fig. 9.

Curves of R_E and R_H against the wall thickness parameter T_A for air-filled apertures are given in [7] for a selection of aspect ratios. For large T_A, R_H and R_E vary as $e^{-\pi T_A}$ and $e^{-\pi T_A \sqrt{1+(A/B)^2}}$ corresponding to the attenuations of the TE_{10} and TM_{11} modes, respectively.

VI. Conclusions

The coupling through small circular and rectangular apertures is separable into electric and magnetic terms, with each type of coupling being associated with a particular set of waveguide mode fields in the aperture region. The concept of aperture polarizability can be extended to include apertures in walls of finite thickness in addition to the well-established zero wall thickness case.

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